

Are oscillons present during a first order electroweak phase transition?

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Abstract

It has been recently argued that localized, unstable, but extremely long-lived configurations, called oscillons, could affect the dynamics of a first order electroweak phase transition in an appreciable way. Treating the amplitude and the size of subcritical bubbles as statistical degrees of freedom, we show that thermal fluctuations are not strong enough to generate subcritical configurations able to settle into a an oscillon long-lived regime.

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Nontopological soliton solutions of classical field theories were introduced a number of years ago by Rosen [1] and their properties have been studied by many authors [2].

Unlike magnetic monopoles and cosmic strings, which arise in theories with non-trivial vacuum topology, nontopological solitons are rendered stable by the existence of a conserved Noether charge carried by fields confined to a finite region of space. The minimum charge of the stable soliton depends upon ratios of coupling constants and in principle can be very small.

Scenarios for actually producing such objects in the early Universe have also been discussed [3]. In particular, this issue has been recently rekindled in ref. [4] where localized, time-dependent, spherically-symmetric solutions of nonlinear scalar field theories, called oscillons, were studied and shown to be, although unstable, extremely long-lived. Indeed, their lifetimes can be of order of $(10^3 - 10^4) m^{-1}$, where m is the mass of the scalar field, *i.e.* much longer than that for a configuration obeying the Klein-Gordon equation for a free scalar field, of order of $5 m^{-1}$.

Oscillons naturally appear during the collapse of spherically symmetric field configurations: if a bubble is formed at rest at the time $t = 0$ with, for example, a "Gaussian" shape

$$\phi(\mathbf{x}, 0) = a e^{-|\mathbf{x}|^2/R^2}, \quad (1)$$

i.e. with an amplitude a at its core and initial radius R , after having radiated most of its initial energy, the bubble settles into a quite long-lived regime, before disappearing by quickly radiating away its remaining energy.

The conditions required for the existence of the oscillons are, apart from having the initial energy above a plateau energy, essentially two [4]: *i)* the initial amplitude of the field at the core needs to be above the inflection point of the potential in order to probe the nonlinearities of the theory and *ii)* the configuration must have an initial radius R bounded from below. To explain these conditions fairly analitically (conclusions are confirmed numerically), we can follow ref. [4] and consider the potential

$$V(\phi) = \frac{m^2}{2}\phi^2 - \frac{\alpha_0 m}{3}\phi^3 + \frac{\lambda}{4}\phi^4. \quad (2)$$

A solution $\phi(\mathbf{x}, t)$ to the equation of motion has energy

$$E[\phi] = \int d^3\mathbf{x} \left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi) \right]. \quad (3)$$

Since during the oscillon regime the subcritical configuration is characterized by a slowly varying radius, we can model the oscillon by writing

$$\phi(\mathbf{x}, t) = a(t) e^{-|\mathbf{x}|^2/R_*^2}, \quad (4)$$

where the radius R_* is kept fixed (a good approximation supported by numerical analysis).

The equation of motion for $a(t)$ can be linearized writing $a(t) = \bar{a}(t) + \delta a(t)$, so that the fluctuation $\delta a(t)$ satisfies the linearized equation

$$\begin{aligned}\ddot{\delta a} &= -\omega^2(\bar{a}, R_*)\delta a, \\ \omega^2(\bar{a}, R_*) &= \frac{3\sqrt{2}}{4}\lambda\bar{a}^2 - \frac{4\sqrt{6}}{9}\alpha_0 m\bar{a} + m^2\left(1 + \frac{3}{m^2 R_*^2}\right).\end{aligned}\quad (5)$$

For $\omega^2(\bar{a}, R_*) < 0$ fluctuations about \bar{a} are unstable, driving the amplitude away from its vacuum value and thus avoiding the rapid shrinking of the initial configuration. These are the fluctuations responsible for the appearance of the oscillon and its relative long lifetime [4].

For fixed α_0 , $\omega^2(\bar{a}, R_*)$ does have a minimum for $\bar{a}_{\min} \simeq 0.51 (\alpha_0 m/\lambda)$, hence oscillons are possible only for

$$R_* > R_{\min} \simeq \sqrt{3/(0.28\alpha_0^2 - \lambda)} \lambda^{1/2} m. \quad (6)$$

For $R_* > R_{\min}$, $\omega^2(\bar{a}, R_*)$ will be negative for amplitudes

$$\bar{a}_- < \bar{a} < \bar{a}_+, \quad (7)$$

where

$$\bar{a}_{\pm} = \frac{8\sqrt{3}\alpha_0 m}{27\lambda} \pm \frac{\sqrt{2}}{3} \left[\frac{96\alpha_0^2 m^2}{81\lambda^2} - 3\sqrt{2} \left(1 + \frac{3}{m^2 R_*^2}\right) \frac{m^2}{\lambda} \right]^{1/2}. \quad (8)$$

In the limit of very large R_* , $R_* \gg 1/m$, \bar{a}_- becomes independent from R_* . To give a numerical example, in the degenerate case $\alpha_0 = (3/\sqrt{2}) \lambda^{1/2}$

$$\bar{a}_{\inf} \simeq 0.3 \frac{m}{\sqrt{\lambda}} < \bar{a}_- \simeq 0.6 \frac{m}{\sqrt{\lambda}} < \bar{a}_{\max} \simeq 0.7 \frac{m}{\sqrt{\lambda}}, \quad (9)$$

where we have indicated with \bar{a}_{\inf} and \bar{a}_{\max} the inflection point closest to $\bar{a} = 0$ and the maximum of the potential $V(\bar{a})$, respectively.

It is then clear why oscillons can form only if their initial amplitude at the core is above the inflection point a_{\inf} and why their initial radius R cannot be too small in order to feel the nonlinearities of the potential.

One of the motivations for studying the evolution of unstable spherically-symmetric configurations comes from the original papers analyzing the role subcritical bubbles may play in the dynamics of weak first order phase transitions [5]. Considering models with double-well potentials in which the system starts localized on one minimum, for sufficiently weak transitions correlation-volumes bubbles of the other phase could be thermally nucleated, giving rise to an effective phase mixing between the two available phases before the reaching from above of the tunneling temperature at which critical bubbles are expected to be nucleated. This could have dramatic consequences for any electroweak baryogenesis mechanisms [6].

Although the presence of thermal fluctuations in any hot system is undisputed, their role in the dynamics of weakly first order phase transitions is still under debate [7]. However, it is clear that, *if* thermally nucleated, long-lived oscillons could appreciably affect the dynamics of a weak first order phase transition at the weak scale. Although their lifetime is small in comparison with the expansion time-scale for temperatures $T \sim 100$ GeV, if oscillons are produced in large enough numbers, their presence will substantially increase the equilibrium number-density of subcritical bubbles of the broken phase. This could effectively make the transition weaker than what predicted from the effective potential. Also, instabilities on the expanding critical bubble walls could be generated by collisions with oscillons, implying that the usual assumption of spherical evolution of the walls may be incorrect.

The aim of this Letter is to investigate whether oscillons can be really present at the onset of a first order electroweak phase transition, *i.e* if subcritical bubbles with initial amplitude at their core and initial radius R satisfying the above conditions *i)* and *ii)* can be thermally nucleated and affect the usual picture of the phase transition dynamics.

To answer this question we will treat both the initial amplitude and the size of subcritical bubble as statistical degrees of freedom along the same lines of what done by Enqvist *et al.* in ref. [7].

First order phase transition and bubble dynamics in the Standard Model have lately been studied in much detail, and it has become increasingly clear [8] that for Higgs masses considerably heavier than 60 GeV, the electroweak phase transition is only of weakly first order. For Higgs mass $M_H > 100$ GeV the calculations, both perturbative and lattice ones, confront technical problems and it is conceivable that for such large Higgs masses the electroweak phase transition is close to a second order and does not proceed by critical bubble formation. Therefore, in this Letter we use a phenomenological Higgs potential for the order parameter ϕ suitable for a simple description of a first order phase transition:

$$V(\phi, T) = \frac{1}{2}m^2(T)\phi^2 - \frac{1}{3}\alpha T\phi^3 + \frac{1}{4}\lambda\phi^4. \quad (10)$$

The properties of the oscillons for the potential (10) can be easily derived from the analysis made for the potential (2) with the substitution $\alpha_0 m \rightarrow \alpha T$. Namely, the oscillon stage can be obtained only if subcritical configurations have initial amplitude greater than the inflection point ϕ_{inf} and sufficiently large size. We also expect that, when increasing λ , the minimum necessary value for R increases, whereas the smallest available value of the amplitude at the core $\bar{\phi}_-$ decreases [4].

When discussing oscillons one has to be sure that initial configurations, which eventually will give rise to an oscillon, are not critical bubbles. Indeed, for the potential (10) and at tunneling temperature T_f , critical bubbles become solutions of the equation

of motion: if they are nucleated with an initial radius R_c (or larger) they can grow converting the metastable phase $\phi = 0$ into the stable phase with lower energy.

Most of the dynamical properties of the electroweak phase transition associated with the potential Eq. (10), such as the smallness of the latent heat, the bubble nucleation rate and the size of critical bubbles, have been discussed extensively in [9]. For the purposes of the present paper it suffices to recall only some of the results.

Assuming that there is only little supercooling, as seems to be the case for the electroweak phase transition, the bounce action can be written as

$$S/T = \frac{\alpha}{\lambda^{3/2}} \frac{2^{9/2} \pi}{3^5} \frac{\bar{\lambda}^{3/2}}{(\bar{\lambda} - 1)^2}, \quad (11)$$

where $\bar{\lambda}(T) = 9\lambda m^2(T)/(2\alpha^2 T^2)$. The cosmological transition temperature is determined from the relation that the Hubble rate equals the transition rate $\propto e^{-S/T}$, yielding $S/T_f \simeq \ln(M_{\text{Pl}}^4/T_f^4) \simeq 150$, where T_f is the transition temperature. Thus we obtain from Eq. (11)

$$\bar{\lambda}(T_f) \simeq 1 - 0.0442 \frac{\alpha^{1/2}}{\lambda^{3/4}} \equiv 1 - \delta. \quad (12)$$

On the other hand, small supercooling implies that $1 - \bar{\lambda} = \delta \ll 1$, i.e. $\alpha \ll 500\lambda^{3/2}$. Solving for $\bar{\lambda}$ in Eq. (11) yields the transition temperature T_f . One finds

$$m^2(T_f) = \frac{2\alpha^2}{9\lambda} \bar{\lambda}(T_f) T_f^2. \quad (13)$$

The extrema of the potential are given by

$$\phi_{\text{min,max}}(T) = \frac{\alpha T}{2\lambda} \left(1 \pm \sqrt{1 - 8\bar{\lambda}/9} \right). \quad (14)$$

Expanding the potential at the broken minimum $\phi_{\text{min}}(T)$ we find

$$-\epsilon \equiv V(\phi_{\text{min}}, T_f) = \frac{1}{6} m^2(T_f) \phi_{\text{min}}^2 - \frac{1}{12} \lambda \phi_{\text{min}}^4 = -0.00218 \frac{\alpha^{9/2}}{\lambda^{15/4}} T_f^4 + \mathcal{O}(\delta^2). \quad (15)$$

The height of the barrier is situated at $\phi_{\text{max}} \simeq \phi_{\text{min}}/2$ with $V(\phi_{\text{min}}, T_c) \equiv V_{\text{max}} = \alpha^4 T_c^4 / (144 \lambda^3)$, where T_c is the temperature at which $V(0) = V(\phi_{\text{min}})$, given by the condition $m(T_c)^2 = (2 \alpha^2 T_c^2 / 9 \lambda)$. As $T_c \simeq T_f$ we may conclude that the thin wall approximation is valid if $-\epsilon/V_{\text{max}} = 0.314 \alpha^{1/2} / \lambda^{3/4} \ll 1$, or $\alpha \ll 10\lambda^{3/2}$. Thus the small supercooling limit is clearly satisfied if the thin wall approximation is valid.

The closest inflection point to $\phi = 0$ at $T \simeq T_c$ is given by

$$\phi_{\text{inf}} \simeq 0.42 \phi_{\text{max}}. \quad (16)$$

To get the size of the critical bubble we still need the surface tension. One easily finds

$$\sigma = \int_0^\infty d\phi \sqrt{2V(T_c)} = \frac{2\sqrt{2}\alpha^3}{91\lambda^{5/2}} T_c^3. \quad (17)$$

We define the critical bubble radius by extremizing the bounce action. The result is

$$R_c = 13.4 \frac{\lambda^{3/4}}{\alpha^{1/2} m(T_f)}. \quad (18)$$

Therefore R_c is much larger than the correlation length $\xi(T_f) = 1/m(T_f)$ at the transition temperature, as it should.

Let us first make the general observation that it is the actual transition temperature T_f rather than the critical temperature T_c which is relevant for the study of oscillons. This is true in the sense that if oscillons are not important at T_f , they most certainly will not be so at T_c . As we shall show, it actually turns out that oscillons are not present even at T_f . This justifies, in retrospect, our choice $T = T_f$ for performing the calculations.

In the case of a weak first order phase transition the critical bubble is typically well described by a thin wall approximation, where the configuration has a flat 'highland' (with ϕ determined by the non-zero minimum of the potential) and a steep slope down to $\phi = 0$. Therefore it seems natural that also a large subcritical bubble should resemble the critical one, *i.e.* when R increases, the form of the subcritical bubble should deform smoothly so that, when $R = R_c$, the bubble is a critical one.

Motivated by this observation, let us define a subcritical bubble as a functional of both the amplitude a and the radius R . For this purpose one has first to study the behaviour of the potential as a function of the amplitude. At T_f there is a interval $\phi \in [a_-, a_+]$ where $V(\phi) \leq 0$. If the amplitude of the bubble is in that interval, there exists a critical bubble-solution of the bounce action. This means that we have a relation $R_c = R_c(a)$ which reproduces Eq. (18) if $a = \phi_{\min}$. Therefore $R_c(a)$ serves as an upper limit for the initial radius R of a subcritical bubble in that region: if $R > R_c(a)$ we exclude such a configuration since it should give rise to a critical bubble and not, eventually, to an oscillon with finite lifetime.

These considerations lead us to define different Ansätze for various regions in the (a, R) -plane. When $\phi \in [a_-, a_+]$, we use an Ansatz such that when $R \rightarrow R_c(a)$, the field configuration goes towards the thin wall form. For small R we use a simple gaussian configuration. For other values of a we always take a thin-wall like Ansatz. Thus we write for $\phi \in [a_-, a_+]$ and $R \leq R_c(a)$

$$\phi(t, R) = a(t) \left[\frac{R_c(a) - R}{R_c(a)} \phi_g + \frac{R}{R_c(a)} \phi_t \right], \quad (19)$$

where t is the time coordinate¹ and

$$\phi_g(R) = e^{-r^2/R^2}, \quad (20)$$

¹Note, however, that we need not to specify the explicit time evolution of a and R when dealing with statistical averages.

$$\phi_t(R) = 1/(e^{m(r-R)} + 1), \quad (21)$$

$$r = |\mathbf{x}|, \quad (22)$$

Such an Ansatz reproduces the requirement that when $R \rightarrow R_c(a)$, subcritical bubbles should resemble critical ones. In practise the statistical averages depend only weakly on a because the main contribution to them comes from the region of small a and large R . Therefore we assume for simplicity that criticality depends weakly on a and take $R_c(a) = R_c$ to be a constant whenever possible.

For $\phi \notin [a_-, a_+]$ we assume that no gaussian component is present and write simply

$$\phi(t, R) = a(t)\phi_t(R). \quad (23)$$

However, the statistical averages are expected to be quite insensitive of the precise form of the configuration.

These Ansätze can be plugged into the action

$$S[a, R] = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - V(\phi) \right] \quad (24)$$

from which the Lagrangian in terms of the dynamical variables a and R can be extracted. In the practical calculation we have, whenever possible, approximated ϕ_t by the step function. After that is a simple matter to calculate the effective Hamiltonian function H_{eff} of the dynamical variables a and R .

Once we have the Hamiltonian, we may calculate the statistical average of a dynamical variable of the type $F(a, R)$ simply by

$$\langle F(a, R) \rangle = \frac{\int dp_R dp_a da dR F(a, R) e^{-\beta H_{\text{eff}}}}{\int dp_R dp_a da dR e^{-\beta H_{\text{eff}}}}. \quad (25)$$

However, because the effective Lagrangian is of the form

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \begin{pmatrix} \dot{a} & \dot{R} \end{pmatrix} K \begin{pmatrix} \dot{a} \\ \dot{R} \end{pmatrix} - \mathcal{V}, \quad (26)$$

where $K = K(a, R)$ is a symmetric matrix, after the momentum integration the average can be cast into the form

$$\langle F(a, R) \rangle = \frac{\int da dR F(a, R) \sqrt{\det K} e^{-\beta \mathcal{V}}}{\int da dR \sqrt{\det K} e^{-\beta \mathcal{V}}}. \quad (27)$$

The matrix

$$K = 4\pi \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \quad (28)$$

and the pseudopotential \mathcal{V} are given separately for the two regions. For $\phi \in [a_-, a_+]$ we obtain

$$\begin{aligned}
R_c^2 K_{11} &= \Delta^2 R^3 A_2^2 + 2\Delta R^4 B_2^1 + \frac{1}{3}R^5 \\
R_c^2 K_{12} &= 2a\Delta^2 R^2 A_4^2 - a\Delta R^3 A_2^2 + \frac{1}{3}aR^4 + aR^5 mI(mR) \\
&\quad + a\Delta R^3 B_2^1 - aR^4 B_2^1 + 2a\Delta R^3 B_4^1 + a\Delta R^4 mJ_2(mR) \\
R_c^2 K_{22} &= 4a^2\Delta^2 R A_6^2 + a^2 R^5 m^2 I(2mR) + 4a^2\Delta R^3 mJ_4(mR) \\
&\quad + a^2 R^3 A_2^2 - 2a^2 R^3 B_2^1 + \frac{1}{3}a^2 R^3 - a^2\Delta R^2 A_4^2 \\
&\quad - 2a^2 R^4 mJ_2(mR) + 4a^2\Delta R^2 B_4^1 + 2a^2 R^4 mI(mR)
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
\frac{R_c(a)^2}{4\pi}\mathcal{V} &= 2a^2\Delta^2 R A_4^2 + 2a^2\Delta R^3 mJ_3(mR) + \frac{1}{2}a^2 R^5 m^2 I(2mR) \\
&\quad + \frac{1}{2}m^2 a^2 \Delta^2 R^3 A_2^2 + m^2 a^2 \Delta R^4 B_2^1 + \frac{1}{6}m^2 a^2 R^5 - \frac{1}{3}\alpha T \frac{a^3}{R_c(a)} \Delta^3 R^3 A_2^3 \\
&\quad - \alpha T \frac{a^3}{R_c(a)} \Delta^2 R^4 B_2^2 - \alpha T \frac{a^3}{R_c(a)} \Delta R^5 B_2^1 - \frac{1}{9}\alpha T \frac{a^3}{R_c(a)} R^6 \\
&\quad + \frac{1}{4}\lambda \frac{a^4}{R_c(a)^2} \Delta^4 R^3 A_2^4 + \lambda \frac{a^4}{R_c(a)^2} \Delta^3 R^4 B_2^3 + \frac{3}{2}\lambda \frac{a^4}{R_c(a)^2} \Delta^2 R^5 B_2^2 \\
&\quad + \lambda \frac{a^4}{R_c(a)^2} \Delta R^6 B_2^1 + \frac{1}{12}\lambda \frac{a^4}{R_c(a)^2} R^7.
\end{aligned} \tag{30}$$

Note that in Eq. (30) the a -dependence of R_c has to be used explicitly because the critical behaviour is determined from it. For the region where $\phi \notin [a_-, a_+]$ the corresponding functions are given by

$$\begin{aligned}
K_{11} &= \frac{1}{3}R^3 \\
K_{12} &= aR^3 mI(mR) \\
K_{22} &= a^2 R^3 m^2 I(2mR)
\end{aligned} \tag{31}$$

and

$$\frac{1}{4\pi}\mathcal{V} = \frac{1}{2}a^2 R^3 m^2 I(2mR) + \frac{1}{6}m^2 a^2 R^3 - \frac{1}{9}\alpha T a^3 R^3 + \frac{1}{12}\lambda a^4 R^3. \tag{32}$$

A number of shorthand notations have been introduced in the previous equations:

$$\Delta = R_c(a) - R \tag{33}$$

$$A_n^k = \int_0^\infty du u^n e^{-ku^2} = \frac{\Gamma(\frac{n+1}{2})}{2k^{\frac{n+1}{2}}} \tag{34}$$

$$B_n^k = \int_0^1 du u^n e^{-ku^2} \tag{35}$$

$$I(x) = \int_0^1 du u^2 e^{x(u-1)} = \frac{1}{x} - \frac{2}{x^2} + \frac{2}{x^3} - \frac{2}{x^3} e^{-x} \quad (36)$$

$$J_n(x) = \int_0^1 du u^n e^{-u^2+x(u-1)}. \quad (37)$$

The range of integration for R possesses an upper limit given by thermalization. Motivated by the fact that thermal fluctuations can generate configurations with spatial size comparable to the critical bubble radius, which may affect the dynamics of a first order phase transition, the authors of ref. [10] have estimated the lifetime of fluctuations of an on-shell Higgs field with zero momentum ($p_0 = m(T)$, $\mathbf{p} = 0$). This choice reflects the fact that critical bubbles are typically much larger than the interparticle distance $\simeq 1/T$ in plasma. Writing $p_0 \equiv \omega - i\gamma/2$, one finds that the dispersion relation is

$$\omega^2 = |\mathbf{p}|^2 + m^2(T) + \frac{1}{4}\gamma^2, \quad (38)$$

where

$$\gamma = \frac{\text{Im } \Gamma^{(2)}}{\omega}, \quad (39)$$

$\Gamma^{(2)}$ being the two-point function for the Higgs field.

The imaginary part arises at one loop level, but because of kinematical constraints, the two loop contribution is actually dominant in the region of physical couplings. The thermalization rate γ for small amplitude scalar fluctuations and large spatial size, $R \sim |\mathbf{p}|^{-1} \gg \gamma^{-1}$, is estimated [10] to be of the order $\gamma \simeq 10^{-2} T$ near the critical temperature, *i.e.* much larger than the typical first order transition time. This means that all small amplitude fluctuations with size larger than

$$R_{\text{max}} = \mathcal{O}(1/\gamma) \quad (40)$$

will effectively be absent from the mixture of subcritical bubbles and must not be counted in the thermal averages. In practice, the limit Eq. (40) is of the order of few times R_c , depending on the actual value of γ . Even if it is not precisely known, its inclusion in the calculations is important. Without it all statistical averages would be dominated by infinite, infinitesimally small fluctuations. Technically this can be seen from the Eq. (27), where the integrals diverge in the limit $a \rightarrow 0$, $R \rightarrow \infty$. It is important to note that the divergence is not a problem of our Ansatz but merely a more general phenomenon, which seems to be related to the general infra-red instability problems emerging in the calculations of the effective action.

We have computed the average initial radius and the amplitude of fluctuations at $T = T_f$ from Eq. (27) numerically using a cut-off $R_{\text{max}} \simeq 3.3 R_c$ (we have checked numerically that results do not change significantly for different choices of R_{max}).

We have taken $\alpha = 0.048$ and varied λ between 4×10^{-2} and 10^{-1} . For larger values of λ the first order electroweak phase transition is close to a second order and does not proceed by critical bubble formation.

For instance, when our phenomenological potential Eq. (10) is fitted to the two loop result for the effective potential calculated in [11] for the Higgs mass $M_H = 70$ GeV, this yields $\lambda \simeq 0.061$. One can readily verify that the thin wall approximation is valid in the chosen range for λ .

In Fig. 1 we show the ratio between $\langle a^2 \rangle^{1/2}$ and the inflection point $\phi_{\text{inf}}(T_f)$ as a function of λ . This ratio is always of order of 0.5. We have also computed numerically the ratio $\langle a \rangle / \phi_{\text{inf}}(T_f)$ which turns out to lie in the range (0.13–0.17) in the given range for λ . We recall that the oscillon stage can be present only if subcritical bubbles are thermally nucleated with initial amplitude above the inflection point [4].

In Fig. 2 we show $\langle R^2 \rangle^{1/2}$ in units of R_c and we have numerically computed $\langle R \rangle$ to be in the range (1.74–1.58) R_c . Oscillons can be formed only if $R > R_{\text{min}} \simeq 3/m(T_f) \simeq (0.55 - 0.27) R_c$ [4].

Note that, when λ increases, the average amplitude at the core also increases, whereas the smallest available amplitude $\bar{\phi}_-$ to settle into an oscillon stage decreases [4]. However, in spite of this tendency, the average amplitude is always smaller than $\bar{\phi}_-$ for any chosen value of λ .

Thus, even if our results seem to indicate that condition *ii*) is satisfied, *i.e.* subcritical bubbles can be thermally nucleated with sufficiently large average initial radius R to give rise to the oscillon regime, nevertheless initial average amplitudes at the core do not satisfy condition *i*) since they always result to be smaller than the inflection point $\phi_{\text{inf}}(T_f)$.

Thermal fluctuations are certainly present at the onset of the electroweak phase transition, but the most probable subcritical configuration generated around the critical temperature, even if with sufficiently large size, is characterized by an amplitude too small to begin the oscillon stage. From this result we can infer that the dynamics of a weak electroweak first order phase transition is not affected by the presence of long-lived oscillons, suggesting that the electroweak baryogenesis scenarios are still viable to explain the generation of the baryon asymmetry in the early Universe. However, we feel that an important issue deserves further study: relaxation time-scales of subcritical configurations depend on the nature of the stochastic force and the strength of dissipation provided by the surrounding thermal bath and their complete knowledge is needed to decide if degrees of freedom are in equilibrium or not inside the subcritical bubble.

One may realize that this is a crucial question by reminding that the effective potential (10), used to describe the free energy associated to the fluctuations, is usually obtained integrating out fermionic and the bosonic degrees of freedom of the theory.

In performing such a calculation, it is commonly assumed that fermions and bosons do have *equilibrium* distributions with a $\phi(\mathbf{x}, t)$ background dependent mass. This is true only if their interaction times with the background $\phi(\mathbf{x}, t)$ are much smaller than

the typical lifetime of the subcritical bubble.

Since this condition is not always satisfied, a full non-equilibrium approach is needed. The latter, however, seems to confirm, or even strengthen, the results of this paper about oscillons [12].

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Figure captions

Figure 1 The plot of the ratio $\langle a^2 \rangle^{1/2} / \phi_{\text{inf}}(T_f)$ as a function of λ and for $\alpha = 0.048$.

Figure 2 The plot of $\langle R^2 \rangle^{1/2}$ in units of R_c as a function of λ and for $\alpha = 0.048$.